Improved Approximations for the Fringing and Shielded Slab-Line Capacitances

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Abstract —Given the dimensions $w/(b-t)$ and t/b for the slab-line geometry or the dimensions s/b and t/b for the fringing capacitance geometry, it is shown how the capacitances associated with each geometry can be determined accurately from new explicit expressions rather than by a computerized search. The two expansions for the nome q', in terms of the dimensions, which are essential for the calculations, are identical except for signs. Finally, comparisons are made with values of the capacitances obtained by a computerized search.

I. Introduction

A LITTLE OVER fifty years ago, Carter [1], Cockcroft [2], and Lundkvist [3] published papers discussing magnetic, electric, and thermal problems, respectively, which were analyzed by mapping the upper half-plane into a suitable infinite polygon by means of the elliptic integral of the third kind having a complex parameter. In his book on elliptic functions, Bowman [4] treats the mapping properties of the elliptic integral of the third kind when the parameter is real as well as imaginary. Shortly thereafter, Bates [5] used the elliptic integral of the third kind with a real parameter to determine the capacitance of the coaxial transmission line referred to as "slab line," shown in Fig. 1(a). Getsinger [6] extended the work of Cockroft by calculating the excess of the capacitance of the coaxial structure of Fig. 1(b) over that due to the infinite parallel plates for both the even- and odd-mode cases. These excess capacitances are now known as "fringing capacitances." Binns and Lawrenson [7] in their recent book apply the mapping provided by an elliptic integral of the third kind with complex parameter to the problem of finding the magnetic field in the finite slot of an electrical machine. Recently, Riblet [8] has used both types of elliptic integrals of the third kind in an improved approximation for the characteristic impedance of rectangular coaxial line.

The elliptic integral of the third kind maps the upper half t plane of Fig. 2 into the infinite five-sided polygons of the s_r and s_c planes: into the s_r plane when the parameter is real and into the s_c plane when it is complex. Consequently, the dimensions of the figures are given parametrically in terms of the modulus and parameter of the elliptic integral. In any practical application, however, it is the dimensions which are given; therefore, it is necessary to find the modulus and parameter from the given dimensions before proceeding. The authors referred to above have all recognized this problem and determined the independent variables from the given dimensions by interpolation with the help of tables and graphs. Now, of course, it is possible to carry out this inversion by a trial-and-error search for these variables with the help of a digital computer. This is not a simple matter, since the two programs require a detailed knowledge of the theory of elliptic functions and must be written with care in order to achieve sufficient accuracy.

In two short papers, Riblet [9], [10] has obtained expansions for the nome q' directly in terms of the dimensions of the polygons, from which the modulus and parameter of the elliptic integrals can be found immediately. The algorithms used in these papers differ somewhat and the expansions are seemingly unrelated. In this paper, the two expansions for the nome q', which are now shown to be identical except for signs, are given to 11 terms. These

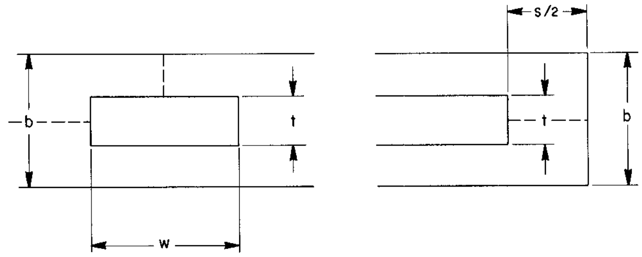

Fig. 1. Geometries for real and complex parameter.

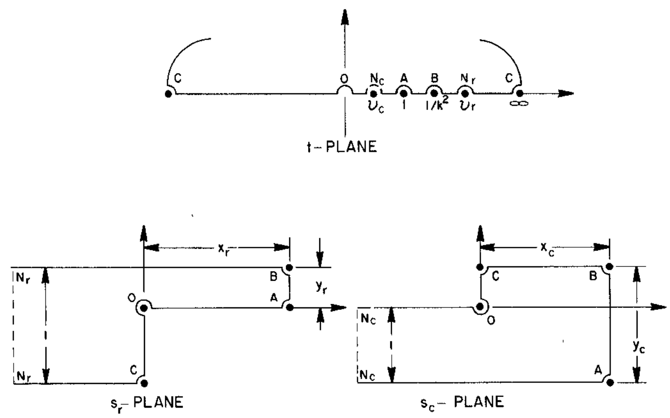

Fig. 2. t, s_r, and s_c planes.

Manuscript received January 4, 1985; revised June 30, 1986.
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IEEE Log Number 8610504.

expansions can then be used to calculate the fringing and slab-line capacitances with great accuracy for most cases of practical interest.

II. The Problem

The Jacobian elliptic integral of the third kind, $\Pi(u, a)$, may be expressed in the form

$$\Pi(u,a)=\frac{\operatorname{cn} a \operatorname{dn} a}{\operatorname{sn} a}\int_0^u \frac{\operatorname{sn}^2 u\,du}{\nu-\operatorname{sn}^2 u} \tag{1}$$

if $\nu = 1/(k^2 \operatorname{sn}^2 a)$. Moreover, if $t = \operatorname{sn}^2 u$, it is readily found that

$$s=\frac{2\Pi(u,a)}{\pi}=\frac{\operatorname{cn} a \operatorname{dn} a}{\pi \operatorname{sn} a}\int_0^t \frac{\sqrt{t}\,dt}{(\nu-t)\sqrt{(1-t)(1-k^2 t)}}. \tag{2}$$

The complex value of s at the points A, B, and C shown in Fig. 2 is given by

$$\begin{aligned} s(A)&=2\Pi(K,a)/\pi\\ s(B)&=2\Pi(K+jK',a)/\pi\\ s(C)&=2\Pi(jK',a)/\pi. \end{aligned} \tag{3}$$

Let us denote a by α_r and ν by ν_r for the real case.[1] Then, if $0<\nu_r<K_r$, $\nu_r<1/k_r^2$ and the upper half of the t plane of Fig. 2 is mapped into the infinite polygon in the s_c plane just below it. For the complex case, if a is replaced by $\alpha_c + jK_c'$, ν is denoted by ν_c, and $0<\alpha_c<K_c$, it is found that $0<\nu_c<1$. Now (2) maps the upper half of the t plane into the infinite polygon shown below it in the s_c plane.

Consideration of Fig. 2 indicates that the x's and y's of both cases satisfy the equations

$$\begin{aligned} x&=\operatorname{Re}\{2\Pi(K,a)/\pi\}\\ y&=1-j2\Pi(jK',a)/\pi. \end{aligned} \tag{4}$$

Then, with the help of [4, p. 82]

$$\begin{aligned} x_r&=\frac{2}{\pi}K_r Z(\alpha_r)\\ y_r&=\frac{2}{\pi}K_r' Z(\alpha_r)+\frac{\alpha_r}{K_r} \end{aligned} \tag{5}$$

and

$$\begin{aligned} x_c&=\frac{2}{\pi}K_c\left\{Z(\alpha_c)+\frac{\operatorname{cn}\alpha_c \operatorname{dn}\alpha_c}{\operatorname{sn}\alpha_c}\right\}\\ y_c&=\frac{2}{\pi}K_c'\left\{Z(\alpha_c)+\frac{\operatorname{cn}\alpha_c \operatorname{dn}\alpha_c}{\alpha_c}\right\}+\frac{\alpha_c}{K_c} \end{aligned} \tag{6}$$

from [11, p. 294, eq. 9]. It is noticed at once, if the elliptic functions are eliminated from (5) and (6), that both pairs of variables satisfy the equation

$$\alpha = Ky - K'x. \tag{7}$$

Since K and K' are functions of the modulus k, this equation expresses the parameter a explicitly in terms of the known dimensions x and y and the modulus. It is useful to write this equation in the form

$$q' = p^{1/y} e^{-\pi x/y} \tag{8}$$

where q' is the complementary nome of elliptic function theory and p is defined by

$$p = e^{-\pi\alpha/K'}. \tag{9}$$

The second equation of (4) provides the additional condition which will permit a determination of k and a explicitly in terms of x and y. For this purpose, the identity

$$\Pi(jK',a)=K'Z(j[K+jK'-a],k') \tag{10}$$

which can be derived from [11, p 423, eq. 1], is useful. Then, for the real case, if a is replaced by α_r in (4)

$$j\frac{2K_r'}{\pi}Z(j[K_r+jK_r'-\alpha_r],k_r')=1-y_r \tag{11}$$

while for the complex case, when a is replaced by $\alpha_c + jK_c'$

$$j\frac{2K_c'}{\pi}Z(j[K_c-\alpha_c],k_c')=1-y_c. \tag{12}$$

The equation obtained for real a is not identical to that obtained for complex a; but, as will be shown, this difference requires only a sign change in the series that expresses α_r as a function of k_r and the dimensions x_r and y_r.

Stated in a general way, (11) is solved for α_r in terms of k_r in series form. This series is then substituted in (8) to eliminate α_r and give k_r in terms of x_r and y_r. It is then possible to find α_r by substituting this value of k_r into (7). It is then shown that, except for signs, the same solutions would be obtained from (12) for k_c and α_c.

In the discussion to follow, it will simplify the notation if the subscript r is ignored when a is real.

From [11, p. 295, eq. 2]

$$j\frac{2K'}{\pi}Z(j[K+jK'-\alpha],k') = -4q'\frac{\sinh n - 2q'^3\sinh 2n + 3q'^8 \sinh 3n + \cdots}{1-2q'\cosh n + 2q'^4\cosh 2n - \cdots} \tag{13}$$

where $n = \pi(K + jK' - \alpha)/K'$. If $p = \exp(-\pi\alpha/K')$, it is found from (11) and (13) that

$$\frac{2p-2q'^2p^{-1}+4q'^2p^2-4q'^6p^{-2}+\cdots+2i\left(q'^{i^2-i}p^{i}-q'^{i^2+i}p^{-i}\right)+\cdots}{1+p+q'^2p^{-1}+q'^2p^2+q'^6p^{-2}+\cdots+\left(q'^{i^2-i}p^{i}+q'^{i^2+i}p^{-i}\right)+\cdots}=1-y. \tag{14}$$

Then, neglecting all terms in q' of i^2+i degree or higher,

[1] Here, the subscripts r and c are used to distinguish the real case from the complex case.

the following equation of degree $2i-1$ in p is obtained:

$$(y+2i-1)q'^{i^2-i}p^i+\cdots+(y+5)q'^6p^3+(y+3)q'^2p^2+(y+1)p+(y-1)+(y-3)q'^2p^{-1}+(y-5)q'^6p^{-2}+\cdots+(y-2i+1)q'^{i^2-i}p^{-i+1}=0. \tag{15}$$

If all terms in q' of 30th degree or higher are neglected, (15) reduces to

$$(y+9)q'^{20}p^5+(y+7)q'^{12}p^4+(y+5)q'^6p^3+(y+3)q'^2p^2+(y+1)p+(y-1)+(y-3)q'^2p^{-1}+(y-5)q'^6p^{-2}+(y-7)q'^{12}p^{-3}+(y-9)q'^{20}p^{-4}=0. \tag{16}$$

It has been found that this equation can be satisfied identically for all terms as high as q'^{20} by the proper selection of the coefficients in the expansion

$$p=a_0+a_1q'^2+\cdots+a_{10}q'^{20}. \tag{17}$$

For example, if terms containing the factor q'^2 are ignored, (16) reduces to

$$(y+1)p+y-1=0 \tag{18}$$

so that

$$a_0=\frac{1-y}{1+y}. \tag{19}$$

Then, if terms involving q'^6 are ignored, (16) reduces to

$$(y+3)q'^2p^3+(y+1)p^2+(y-1)p+(y-3)q'^2=0. \tag{20}$$

Substitution of (17) in (20) yields, when the coefficients of q'^2 and q'^4 are equated to zero, respectively,

$$a_1=\frac{3-y-(3+y)a_0^3}{a_0(1+y)} \tag{21}$$

and

$$a_2=\frac{3(y+3)a_0^2a_1+(y+1)a_1^2}{a_0(1+y)}. \tag{22}$$

In fact, each higher power in q'^2 yields an additional equation in which the new unknown occurs to the first power with a nonzero coefficient. Thus, each of the unknowns in (17) can be expressed as a rational function of the unknowns determined previously and, hence, as a rational function of y. When the first three terms of (17) are found in this way by algebraic substitution

$$p=\frac{1-y}{1+y}\left\{1+\frac{16y}{(1-y^2)^2}q'+\frac{16y}{(1-y^2)^4}\cdot(3y^4-18y^2+8y-9)q'^4+\cdots\right\}. \tag{23}$$

The only difference in the discussion when a is complex arises from the jK', which occurs in (11) but is not present in (12). This means that $p_c=\exp(-\pi a_c/K_c')$, except for a change in sign, will satisfy an equation identical to (14) with y replaced by y_c. Moreover, it is clear from Fig. 2 that $y<1<y_c$. Then, since the leading coefficient in both p and p_c is positive, they can be expressed in the form

$$p=\left|\frac{1-y}{1+y}\right|\left\{1+\frac{16y}{(1-y^2)^2}q'^2+\frac{16y}{(1-y^2)^4}\cdot(3y^4-18y^2+8y-9)q'^4+\cdots\right\}. \tag{24}$$

The further results of this paper depend only on (8) and (24) and so are valid for a real or complex parameter a. The terms on the right-hand side of (24) depend only on k and y so that the substitution of p into (8), eliminates a and gives an expression for k entirely in terms of x and y.

The coefficients in

$$p^{1/y}=b_0+b_1q'^2+\cdots+b_iq'^{2i}+\cdots \tag{25}$$

follow directly from those of (24) with the help of the binomial theorem. By algebraic substitution, it is found that

$$p^{1/y}=\left|\frac{1-y}{1+y}\right|^{1/y}\left\{1+\frac{16}{(1-y^2)^2}q'^2+\frac{16}{(1-y^2)^4}(3y^4-18y^2-1)q'^4+\cdots\right\}. \tag{26}$$

Then, if an expansion of the form

$$q'=c_0e^{-\pi x/y}+c_1e^{-3\pi x/y}+\cdots+c_ie^{-2i\pi x/y}+\cdots \tag{27}$$

is assumed and (25) and (27) are substituted into (8), the values of $c_0,\cdots,c_i$ can be determined. In fact, $c_0=b_0$, $c_1=b_1c_0^2$, $c_2=2b_1c_0c_1+b_2c_0^4$ and, in general, each coefficient in (27) can be expressed as a multinomial in the previously determined coefficients of (25) and (27). When the required substitutions are carried through

$$q'=|1-y^2|\left\{\left|\frac{1-y}{1+y}\right|^{1/y}\frac{1}{|1-y^2|}e^{-\pi x/y}+\left|\frac{1-y}{1+y}\right|^{3/y}\frac{16}{|1-y^2|^3}e^{-3\pi x/y}+\left|\frac{1-y}{1+y}\right|^{5/y}\frac{16(3y^4-18y^2+31)}{|1-y^2|^5}e^{-5\pi x/y}+\cdots\right\}. \tag{28}$$

Performing the algebraic steps of this procedure, even to find the coefficient of $\exp(-5\pi x/y)$ in (28), is tedious. The regular appearance of the coefficients suggests, however, that their polynomial factors can be determined from their values at a number of points on the real axis. When this was implemented on a digital computer, it was found that

$$q'=|1-y^2|\sum_{i=0}^{10}C(i)T^{2i+1} \tag{29}$$

where

$$T=\left|\frac{1-y}{1+y}\right|^{1/y}\frac{e^{-\pi x/y}}{|1-y^2|}$$

and

$$C(0)=1$$

$$C(1)=16$$

$$C(2)=16(3y^4-18y^2+31)$$

$$C(3)=64(3y^8-72y^6+450y^4-1064y^2+875)/3$$

$$C(4)=16(7y^{12}-270y^{10}+3693y^8-22196y^6+64689y^4-90078y^2+48251)$$

$$C(5)=32(45y^{16}-4320y^{14}+104460y^{12}-1150560y^{10}+6733950y^8-22178400y^6+41111724y^4-39954400y^2+15829021)/15$$

$$C(6)=64(1485y^{20}-133650y^{18}+4888125y^{16}-85539960y^{14}+826058970y^{12}-4762247820y^{10}+17009930850y^8-37841542200y^6+50880292793y^4-37758531250y^2+11856583937)/495$$

$$C(7)=128(7y^{24}-1512y^{22}+81774y^{20}-2081128y^{18}+29835057y^{16}-263259920y^{14}+1502298980y^{12}-5687848656y^{10}+14378327929y^8-23929259528y^6+25111900366y^4-15033150664y^2+3907837359)/7$$

$$C(8)=16(4725y^{28}-878850y^{26}+61521075y^{24}-2136973860y^{22}+42549625125y^{20}-530922866670y^{18}+4388387507595y^{16}-24858706628280y^{14}+98340298079079y^{12}-273370988493246y^{10}+530402371431681y^8-701721204772836y^6+602202601548415y^4-301697220375698y^2+66889468440385)/315$$

$$C(9)=16(36855y^{32}-10886400y^{30}+1044618120y^{28}-47830608000y^{26}+1261417665060y^{24}-21103362241920y^{22}+237498114019320y^{20}-1867896202394880y^{18}+10522731466801386y^{16}-43079389762268160y^{14}+128901200546148408y^{12}-281029242225518208y^{10}+440413731938457828y^8-482195108654477696y^6+349472852081130696y^4-150411589033214976y^2+29067276705016727)/2835$$

$$C(10)=32(675y^{36}-214650y^{34}+24524775y^{32}-1416164400y^{30}+47753927100y^{28}-1030782763800y^{26}+15126283744860y^{24}-157157532680400y^{22}+1188621307767210y^{20}-6667418727685020y^{18}+28058027730058290y^{16}-89039060380434000y^{14}+212835395264864940y^{12}-380072612372777880y^{10}+498218768602080812y^8-464481620112148400y^6+291058626317465931y^4-109747648387208250y^2+18794513408019807)/75.$$

Tedious algebra was involved in finding the first seven coefficients in (29). Explicit formulas expressing the first seven of the coefficients in (17), (25), and (27) in terms of coefficients already known were derived. In fact, the coefficients in (24), (26), and (28) were obtained by detailed substitution in the simpler of these formulas. To find the next four coefficients, these formulas were programmed on a digital computer and the values of each unknown coefficient were found successively at different values of y on the real axis. The polynomial factor of each of the unknown coefficients was then reconstructed from these values.

A method which avoided the tedious algebra involved in expressing explicitly the unknown coefficient in terms of known coefficients was adopted to find the last four coefficients in (29). It can be shown that (16) permits the determination of the value of the coefficients of q'^{2i} in terms of the values of the coefficients of lower degree occurring in the powers of p, up to the ninth, so long as $2i < 30$. Since these values depend only on the values of the coefficients of degree $< 2i$, it follows by induction that the value of the coefficient of q'^{2i} can be found directly from (16) for any value of y if $2i < 30$.

The convergence of the expansion for q' has not been demonstrated. It is possible, however, that it converges over a wide range of values of the dimensions for both geometries. In spite of the large values of the coefficients in the highest degree terms, there is sufficient cancellation in them so that when they are multiplied by $(1-y)$ to the proper power for $y<1$ and divided by $(1+y)$ to the proper power for $y>1$, the final values are small, uniformly decreasing quantities for $y<1$ and small, generally decreasing quantities of changing sign for $y>1$.

In any case, the values of k and a obtained from (29) are extremely accurate. For x/y as small as 0.05, the values of k and a obtained from (29) are accurate within 0.01% for values of y corresponding to values of t/b ranging from 0.05 to 0.9. For values of $x/y=0.2$, agreement to ten decimal places is found. As x/y increases, the number of terms of (29) required for a given accuracy decreases, as one might expect. For example, when $x/y = 0.8$, four terms of (29) give ten-place accuracy; when $x/y = 1.6$, only three terms are sufficient for nine-place accuracy.

Equation (29) determines the value of q' directly in terms of the values of the dimensions x and y of Fig. 2. For the geometry of Fig. 2(a), $y<1$ and for the geometry of Fig. 2(b), $y>1$. In either case, k is given by the well-known expression

$$\sqrt{k} = \frac{1-2q'+2q'^4-2q'^9+\cdots}{1+2q'+2q'^4+2q'^9+\cdots}. \tag{30}$$

When a is real, the geometry is that of a slab line. Then

$$x = x_r = w/b$$
$$y = y_r = 1 - t/b. \tag{31}$$

For given dimensions, the modulus k_r is first determined from (31), (29), and (30). It is then possible to find $a = \alpha_r$ from (7) since K_r and K_r' are now known. The capacitance C_s of the structure is then given by the expression

$$C_s = 4K_0'/K_0. \tag{32}$$

Here, K_0 and K_0' are the complete elliptic integrals of the first kind of modulus k_0, where

$$k_0' = \frac{\operatorname{cn}(\alpha_r, k_r)}{\operatorname{dn}(\alpha_r, k_r)}. \tag{33}$$

When a is complex, the geometry is that associated with the approximate fringing capacitance C_{f_0}'. Then

$$x = x_c = s/(b-t)$$
$$y = y_c = b/(b-t). \tag{34}$$

As above, k_c and α_c are found from (34), (29), (30), and (7).

Explicit formulas for the C_{f_0}' have been given by Cockcroft [2], Getsinger [3], and Riblet [12]. In the expansion for C_{f_0}' given in [12, eq. (4)] a must be replaced by $K - \alpha_c$. Moreover, after this substitution, the $Z(a)$ can be eliminated by introducing the value of x from (6). Finally

$$C_{f_0}' = \frac{\alpha s}{K(b-t)} - 2\log\left(k \operatorname{sn}\alpha \operatorname{cn}\alpha\theta_n^2(\alpha)\right)/\pi. \tag{35}$$

Here, $\theta_n(\alpha) = \theta(\alpha)/\theta(0)$ and the subscript c has been omitted.

Table I compares the values of C_s and C_{f_0}' obtained from (32) and (35) using (29) with values obtained from an accurate computerized search. The values in the top row are the accurate values, while the next row gives the values obtained by using all 11 of the terms of (29). The third row uses only seven of the terms of the series. The values in the table show that the use of all 11 terms of the expansion substantially increases the accuracy with which C_s and C_{f_0}' are approximated for small values of s/b and $w/(b-t)$. For values of s/b and $w/(b-t) > 0.2$, the number of terms required for ten-place agreement decreases rapidly, as has been observed. It is clear that, for most geometries of practical interest, (29) will determine both the slab-line capacitance C_s and the approximate fringing capacitance C_{f_0}' with great accuracy, enabling the previous computerized search methods to be avoided.

TABLE I
VALUES OF C_{f_0}' AND C_s

	C'_{f0}			$C_s/4$		
	s/b			w/(b-t)		
t/b	05	10	20	05	10	20
05	2 680566230 2 68104 2 708	1 789173248 1 78917345 1 78937	1 191716807 1 191716807 1 191716853	5072294390 50723024 507272	5792213317 5792213317 57922207	7002360012 7002360012 7002360015
10	3 649045900 3 649090 3 682	2 261324192 2 261324203 2 26151	1 422309070 1 422309070 1 422309113	5788907107 57889129 578926	6465702057 6465702061 64657081	7638956236 7638956236 7638956238
50	11 31782862 11 317839 11 3197	5 970222677 5 970222680 5 9702275	3 203973057 3 203973057 3 203973057	1 071929709 1 07192998 1 071948	1 131164070 1 131164070 1 13116437	1 240413846 1 240413846 1 240413846
90	18 64353725 18 643611 18 6475	9 558728800 9 558728828 9 558760	5 143537249 5 143537250 5 143537260	2 128467967 2 12846820 2 128484	2 186305706 2 186305706 2 18630596	2 294119392 2 294119392 2 294119392

III. EVALUATION OF THE ELLIPTIC FUNCTIONS

The determination of C_s and C_{f_0}' from the values of k and a found by a computerized search requires routines for calculating values of various elliptic functions. The same is true of the values of C_s and C_{f_0}' obtained from the k and a found from (29), (30), and (7). Thus, an error somewhere in these routines could result in an error in the first row for each t/b parameter given in Table I without affecting the agreement between the accurate first-row values and the approximate second- and third-row values. In order to minimize this possibility, two routines for evaluating the elliptic functions were employed. The values of the elliptic functions used in evaluating C_s and C_{f_0}' in terms of the k and a found by a computerized search were determined by means of the q series for the theta functions, while Landen's transformation was used to find the approximate values of C_s and C_{f_0}'.

ACKNOWLEDGMENT

The author is indebted to the reviewers for suggestions which materially simplify and clarify the arguments and the statement of the results of this paper. In particular, the x and y notation which is used here in place of the w, s, b, and t notation of Fig. 1 has the consequence that the two equations that were required to express the sense of (7) can be written as one. Moroever, the absolute value signs in (24) make it possible to express q' by a single expansion in y and x/y rather than as two series with terms differing only in sign.

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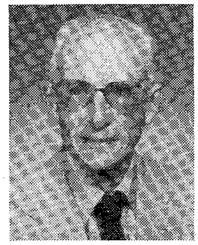

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From 1939 to 1941, he taught mathematics at Adelphi College, Garden City, NY, and at Hofstra College, Hempstead, NY. He joined the staff of the Massachusetts Institute of Technology Radiation Laboratory, Cambridge, in 1942, and at the close of World War II was in charge of one of the three development sections of the Antenna Group. From 1946 to 1948, he headed the RF group at the Submarine Signal Company, Boston, MA. At present he is affiliated with the Microwave Development Laboratories, Inc., Needham Heights, MA.

Dr. Riblet is a member of the American Mathematical Society. In 1976, he was awarded the 1975 Microwave Career Award by the Professional Group, Microwave Theory and Techniques.